Mathematics Level 1 and Level 2

One-hour subject tests

Purpose

- Measure knowledge of mathematics through the first three years of college-preparatory mathematics for Level 1 and through precalculus for Level 2

Format

- 50 multiple-choice questions
- Questions covering content typically taught in the first three years (for Level 1) or four years (for Level 2) of college-preparatory mathematics (Students are not expected to have studied every topic on either test.)

Recommended Preparation

Mathematics Level 1

- Three years of college-preparatory mathematics, including two years of algebra and one year of geometry

Mathematics Level 2

- More than three years of college-preparatory mathematics, including two years of algebra, one year of geometry, and elementary functions (precalculus) and/or trigonometry.
- If you have had preparation in trigonometry and elementary functions and have attained grades of B or better in these courses, select Level 2. If you are sufficiently prepared to take Level 2 but take Level 1 in hopes of receiving a higher score, you may not do as well as you expect.

Score

- Total score: 200–to–800 scale.
- Because the content measured by the two tests differs considerably, you cannot use your score on one test to predict your score on the other or to compare scores.

CONTENT	Approximate % of Test	
Topics Covered*	**Level 1**	**Level 2**
Number and Operations operations, ratio & proportion, complex numbers, counting, elementary number theory, matrices, sequences, *series, vectors*	10–14	10–14
Algebra and Functions expressions, equations, inequalities, representation and modeling, properties of functions (linear, polynomial, rational, exponential, *logarithmic, trigonometric, inverse trigonometric, periodic, piecewise, recursive, parametric*)	38–42	48–52
Geometry and Measurement	38–42	28–32
Plane Euclidean/Measurement	18–22	-------
Coordinate lines, parabolas, circles, *ellipses, hyperbolas,* symmetry, transformations, *polar coordinates*	8–12	10–14
Three-dimensional solids, surface area & volume (cylinders, cones, pyramids, spheres, prisms), *coordinates in three dimensions*	4–6	4–6
Trigonometry right triangles, identities, *radian measure, law of cosines, law of sines, equations, double angle formulas*	6–8	12–16
Data Analysis, Statistics, and Probability mean, median, mode, range, interquartile range, *standard deviation,* graphs and plots, least-squares regression (linear, *quadratic, exponential*), probability	6–10	6–10

** Topics in italics are tested on the Level 2 Test only. The content of Level 1 overlaps somewhat with that on Level 2, but the emphasis on Level 2 is on more advanced content. Plane Euclidean geometry is not tested directly on Level 2.*

Geometric Figures

Figures that accompany problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a particular problem that the figure is not drawn to scale. Even when figures are not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. The text "Note: Figure not drawn to scale." is included on the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally.

Calculators

Some questions on these tests cannot be solved without a scientific or a graphing calculator. You do not need to use a calculator to solve every question, but it is important to know when and how to use one. If you take these tests without a calculator, you will be at a disadvantage.

- **We recommend the use of a graphing calculator over a scientific calculator because a graphing calculator may provide an advantage on some questions.**
- You may not use a calculator for other Subject Tests and must put it away when not taking a mathematics test.

For 50–60 percent of the questions on Level 1 and 35–45 percent of the questions on Level 2, there is no advantage, perhaps even a disadvantage, to using a calculator. For 40–50 percent of the questions on Level 1 and 55–65 percent of the questions on Level 2, a calculator may be useful or necessary.

What Type of Calculator Should I Bring?

- Bring a calculator that you are used to using. It may be a scientific or a graphing calculator. If you're comfortable with both a scientific and a graphing calculator, bring a graphing calculator.
- Verify that your calculator is in good working condition before you take the test. You may bring batteries and a backup calculator to the test center.
- No substitute calculators or batteries will be available at the test center. Students may not share calculators.
- If your calculator malfunctions during the Level 1 or Level 2 Tests and you do not have a backup calculator, you must tell your test supervisor when the malfunction occurs in order to cancel scores on these tests only.

The following are not permitted:

- Laptops and portable/handheld computers
- Electronic writing pads or pen-input/stylus-driven devices (e.g., Palm, PDAs, Casio ClassPad 300)
- Pocket organizers
- Models with QWERTY (i.e., typewriter) keypads (e.g., TI-92 Plus, Voyage 200)
- Models with paper tapes
- Models that make noise or "talk"
- Models that require an electrical outlet
- Cell phone calculators

Using the Calculator

- Remember, only some questions on these tests require the use of a calculator. First decide how you will solve a problem; then determine whether the calculator is needed.
- Do not round any intermediate calculations. If you get a result from the calculator for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step, your answer may not be one of the choices.

Sample Questions

All questions in the Mathematics Level 1 and Mathematics Level 2 Subject Tests are multiple-choice questions in which you are asked to choose the best response from the five choices offered. The directions for the tests are below:

Directions: For each of the following problems, decide which is the **BEST** of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) Level 1: The only angle measure used on this test is degree measure. Make sure your calculator is in the degree mode.

Level 2: For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$, where x is in the domain of f.

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

Reference Information. The following information is for your reference in answering some of the questions in this test.

Volume of a right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

Lateral Area of a right circular cone with circumference of the base c and slant height ℓ: $S = \frac{1}{2}c\ell$

Volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

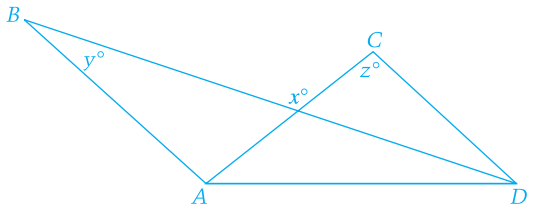
Surface Area of a sphere with radius r: $S = 4\pi r^2$

Volume of a pyramid with base area B and height h: $V = \frac{1}{3}Bh$

Mathematics Level 1

1. A band wants to distribute its music on compact discs (CDs). The equipment to produce the CDs costs $250, and blank CDs cost $5.90 for a package of 10. Which of the following represents the total cost, in dollars, to produce n CDs, where n is a multiple of 10?

   (A) $(250 + 0.59)n$ (B) $250 + 0.59n$
   (C) $(250 + 5.90)n$ (D) $250 + 5.90n$
   (E) $250n + 5.90$



2. In the figure above, $\overline{AB}$ and $\overline{CD}$ are parallel. What is x in terms of y and z?

   (A) $y + z$
   (B) $2y + z$
   (C) $2y - z$
   (D) $180 - y - z$
   (E) $180 + y - z$

3. A number n is increased by 8. If the cube root of that result equals –0.5, what is the value of n?

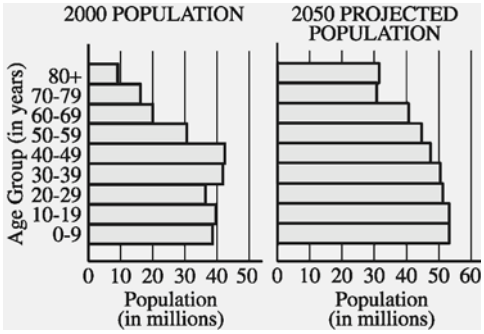
   (A) –15.625
   (B) –8.794
   (C) –8.125
   (D) –7.875
   (E) 421.875

4. If a and b are real numbers, $i^2 = -1$, and $(a + b) + 5i = 9 + ai$, what is the value of b?

   (A) 4 (B) 5 (C) 9
   (D) $4 + 5i$ (E) $5 + 4i$

5. What are all values of x for which $4 - x^2 \geq x - 2$?

   (A) $x \geq -3$ (B) $-5 \leq x \leq 0$
   (C) $-3 \leq x \leq 2$ (D) $x \leq -3$ or $x \geq 2$
   (E) $-2 \leq x \leq 3$



6. The graphs above show United States Census Bureau population figures for the year 2000 for various age groups, together with projections for the year 2050. Of the following age groups, for which is the projected percent increase in population from 2000 to 2050 greatest?

   (A) 30–39 (B) 40–49 (C) 50–59
   (D) 60–69 (E) 70–79

7. If $\log_c a = x$, which of the following must be true?

   (A) $a^c = x$ (B) $a^x = c$ (C) $c^a = x$
   (D) $c^x = a$ (E) $x^c = a$

8. If $f(x) = x + 3$ and $g(x) = \dfrac{x^2 - 9}{x - 3}$, which of the following statements are true about the graphs of f and g in the xy-plane?

I. The graphs are exactly the same.
II. The graphs are the same except when $x = 3$.
III. The graphs have an infinite number of points in common.

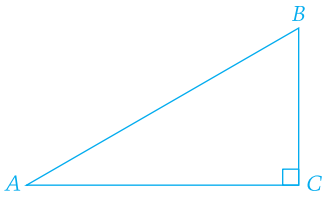
(A) I only (B) II only (C) III only
(D) I and III (E) II and III

9. If line ℓ is the perpendicular bisector of the line segment with endpoints $(2, 0)$ and $(0, -2)$, what is the slope of line ℓ?

(A) 2 (B) 1 (C) 0
(D) -1 (E) -2

10. Twenty students have each sampled one or more of three kinds of candy bars that a school store sells. If 3 students have sampled all three kinds, and 5 have sampled exactly two kinds, how many of these students have sampled only one kind?

(A) 8 (B) 12 (C) 15
(D) 17 (E) 18



Note: Figure not drawn to scale.

11. In the figure above, $\triangle ABC$ has a right angle at C. If the length of side $\overline{AC}$ is 10 and the measure of $\angle BAC$ is 22°, what is the length of side $\overline{BC}$?

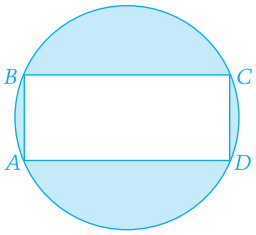
(A) 3.7 (B) 4.0 (C) 5.8
(D) 6.8 (E) 9.3

12. The function h given by $h(t) = -16t^2 + 46t + 5$ represents the height of a ball, in feet, t seconds after it is thrown. To the nearest foot, what is the maximum height the ball reaches?

(A) 5 (B) 23 (C) 35
(D) 38 (E) 46

13. The front, side, and bottom faces of a rectangular solid have areas of 24 square centimeters, 8 square centimeters, and 3 square centimeters, respectively. What is the volume of the solid, in cubic centimeters?

(A) 24 (B) 96 (C) 192
(D) 288 (E) 576



14. Rectangle $ABCD$ is inscribed in the circle shown above. If the length of side $\overline{AB}$ is 5 and the length of side $\overline{BC}$ is 12, what is the area of the shaded region?

(A) 40.8 (B) 53.1 (C) 72.7
(D) 78.5 (E) 81.7

15. If $f(x) = x^4 - 3x^3 - 9x^2 + 4$, for how many real numbers k does $f(k) = 2$?

(A) None (B) One (C) Two
(D) Three (E) Four

16. If the measure of one angle of a rhombus is 60°, then the ratio of the length of its longer diagonal to the length of its shorter diagonal is

(A) 2 (B) $\sqrt{3}$ (C) $\sqrt{2}$
(D) $\dfrac{\sqrt{3}}{2}$ (E) $\dfrac{\sqrt{2}}{2}$

Time t (years)	0	1	2	5
Value $v(t)$ (dollars)	15,000	13,000	10,900	3,000

17. When purchased, an automobile is valued at $15,000. Its value depreciates at the rate shown in the table above. Based on a least-squares linear regression, what is the value, to the nearest hundred dollars, of the automobile when $t = 4$?

(A) $5,400 (B) $5,500 (C) $5,600
(D) $6,400 (E) $7,000

Mathematics Level 2

18. What is the distance in space between the points with coordinates $(-3, 6, 7)$ and $(2, -1, 4)$?

(A) 4.36 (B) 5.92 (C) 7.91
(D) 9.11 (E) 22.25

19. If $f(x) = \dfrac{3x + 12}{2x - 12}$, what value does $f(x)$ approach as x gets infinitely larger?

(A) -6 (B) $-\dfrac{3}{2}$ (C) -1
(D) $\dfrac{2}{3}$ (E) $\dfrac{3}{2}$

20. In January 1990 the world's population was 5.3 billion. Assuming a growth rate of 2 percent per year, the world's population, in billions, for t years after 1990 can be modeled by the equation $P = 5.3(1.02)^t$. According to the model, the population growth from January 1995 to January 1996 was

(A) 106,000,000
(B) 114,700,000
(C) 117,000,000
(D) 445,600,000
(E) 562,700,000

21. What is the measure of one of the larger angles of a parallelogram in the xy-plane that has vertices with coordinates $(2, 1)$, $(5, 1)$, $(3, 5)$, and $(6, 5)$?

(A) 93.4° (B) 96.8° (C) 104.0°
(D) 108.3° (E) 119.0°

22. For some real number t, the first three terms of an arithmetic sequence are $2t$, $5t - 1$, and $6t + 2$. What is the numerical value of the fourth term?

(A) 4 (B) 8 (C) 10
(D) 16 (E) 19

23. The diameter and height of a right circular cylinder are equal. If the volume of the cylinder is 2, what is the height of the cylinder?

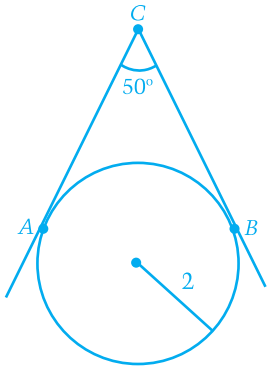
(A) 1.37 (B) 1.08 (C) 0.86
(D) 0.80 (E) 0.68

24. If $\sin\theta = 0.57$, then $\sin(\pi - \theta) =$

(A) -0.57 (B) -0.43 (C) 0
(D) 0.43 (E) 0.57

25. In a group of 10 people, 60 percent have brown eyes. Two people are to be selected at random from the group. What is the probability that neither person selected will have brown eyes?

(A) 0.13 (B) 0.16 (C) 0.25
(D) 0.36 (E) 0.64



26. In the figure above, two lines are tangent to a circle of radius 2 at points A and B. What is the length of segment AB (not shown)?

(A) 1.37 (B) 1.69 (C) 3.06
(D) 3.63 (E) 4

27. If $x - 2$ is a factor of $x^3 + kx^2 + 12x - 8$, then $k =$

(A) -6 (B) -3 (C) 2
(D) 3 (E) 6

28. If $f(x) = \sqrt[3]{x^3 + 1}$, what is $f^{-1}(1.5)$?

(A) 3.4 (B) 2.4 (C) 1.6
(D) 1.5 (E) 1.3

x	−9.8	−0.9	5.2	8.8
y	0.12	2.43	18.46	68.4

29. Which of the following equations best models the data in the table above?

(A) $y = -3.3(1.4)^x$
(B) $y = -1.4(3.3)^x$
(C) $y = 1.4(3.3)^x$
(D) $y = 3.3(1.4)^x$
(E) $y = 1.4x^{3.3}$

$$C = -1.02F + 93.63$$

30. The linear regression model above is based on an analysis of nutritional data from 14 varieties of cereal bars to relate the percent of calories from fat (F) to the percent of calories from carbohydrates (C). Based on this model, which of the following statements must be true?

I. There is a positive correlation between C and F.
II. When 20 percent of calories are from fat, the predicted percent of calories from carbohydrates is approximately 73.
III. The slope indicates that as F increases by 1, C decreases by 1.02.

(A) II only (B) I and II only
(C) I and III only (D) II and III only
(E) I, II, and III

31. A line has parametric equations $x = 5 + t$ and $y = 7 + t$, where t is the parameter. The slope of the line is

(A) $\frac{5}{7}$ (B) 1 (C) $\frac{7+t}{5+t}$
(D) $\frac{7}{5}$ (E) 7

32. What is the range of the function defined by $f(x) = \frac{1}{x} + 2$?

(A) All real numbers
(B) All real numbers except $-\frac{1}{2}$
(C) All real numbers except 0
(D) All real numbers except 2
(E) All real numbers between 2 and 3

33. The number of hours of daylight, d, in Hartsville can be modeled by $d = \frac{35}{3} + \frac{7}{3}\sin\left(\frac{2\pi}{365}t\right)$, where t is the number of days after March 21. The day with the greatest number of hours of daylight has how many more daylight hours than May 1? (March and May have 31 days each. April and June have 30 days each.)

(A) 0.8 hr (B) 1.5 hr (C) 2.3 hr
(D) 3.0 hr (E) 4.7 hr

	Day 1	Day 2	Day 3
Model X	20	18	3
Model Y	16	5	8
Model Z	19	11	10

34. The table above shows the number of digital cameras that were sold during a three-day sale. The prices of models X, Y, and Z were \$99, \$199, and \$299, respectively. Which of the following matrix representations gives the total income, in dollars, received from the sale of the cameras for each of the three days?

(A) $\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} [99 \quad 199 \quad 299]$

(B) $\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} \begin{bmatrix} 99 \\ 199 \\ 299 \end{bmatrix}$

(C) $[99 \quad 199 \quad 299] \begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 99 \\ 199 \\ 299 \end{bmatrix} \begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

(E) $99\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} + 199\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} + 299\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

ANSWERS

The estimated difficulty level, on a scale of 1 to 5, with 1 the easiest and 5 the most difficult, is in parentheses.

Mathematics Level 1

1. B (2)	**5. C (3)**	**9. D (4)**	**13. A (4)**	**17. C (5)**
2. A (2)	**6. D (4)**	**10. B (3)**	**14. C (4)**	
3. C (2)	**7. D (3)**	**11. B (3)**	**15. E (3)**	
4. A (3)	**8. E (3)**	**12. D (4)**	**16. B (5)**	

Mathematics Level 2

18. D (2)	**22. E (4)**	**26. D (4)**	**30. D (4)**	**34. C (3)**
19. E (2)	**23. A (3)**	**27. A (2)**	**31. B (3)**	
20. C (4)	**24. E (3)**	**28. E (4)**	**32. D (3)**	
21. C (4)	**25. A (4)**	**29. D (4)**	**33. A (4)**	